

MATH 3060 Tutorial 6

Chan Ki Fung

October 18, 2022

1. Let (X, d) be a metric space, and fix a point $x_0 \in X$.

- (a) For each $x \in X$, show that the function

$$\rho_x : y \mapsto d(y, x) - d(y, x_0)$$

is a bounded continuous function on X .

- (b) Show that the map $x \mapsto \rho_x$ defines an isometric embedding of X into $C_0(X)$ (the set of bounded continuous function with uniform metric).

- (c) Show that X has a completion.

2. (a) Let $A = (a_{ij})$ be an $n \times n$ matrix with

$$\sum_i |a_{ij}| < 1$$

Show that $I - A$ is invertible.

- (b) Show that the equation

$$3x^3 - 3x^2 + x - 2 = 0$$

has a real root $c \geq 1$. for each j .

3. Consider the function $f : [0, \infty) \rightarrow [0, \infty)$ given by

$$x \mapsto \sqrt{1 + x^2}$$

show that

$$|f(x) - f(y)| < |x - y|$$

for all $x \neq y \in [0, \infty)$, but f has no fixed point.